International Workshop on Advances in Numerical Analysis and Scientific Computation

Shanghai Normal University, Shanghai, China

June 30-July 3, 2018

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Themes and Objectives

The purpose of this conference is to gather experts in numerical analysis and scientific computation to exchange ideas and to discuss future developments and trends of these fields. This international conference is dedicated to the memory of late Professor Ben-yu Guo (1942-2016), who has made outstanding contributions to numerical analysis and scientific computation.

Sponsors

- Shanghai Normal University
- Shanghai University
- Scientific Computing Key Laboratory of Shanghai Universities
- 《Communication on Applied Mathematics and Computation》

Organizing Committee

Zhongci Shi	Chinese Academy of Sciences, China （Chair）
Benqi Guo	University of Manitoba, Canada （Co-Chair）
Heping Ma	Shanghai University, China （Co-Chair）
Hongjiong Tian	Shanghai Normal University, China （Co-Chair）
Jin Cheng	Shanghai University of Finance and Economics, China
Jianguo Huang	Shanghai Jiao Tong University, China
Bendong Lou	Shanghai Normal University, China
Jie Shen	Purdue University, USA and Xiamen University, China
Yangfeng Su	Fudan University, China
Li-Lian Wang	Nanyang Technological University, Singapore
Qing-Wen Wang	Shanghai University, China
Roderick S. C. Wong	City University of Hong Kong, Hong Kong
Xuejun Xu	Tongji University, China
Yuesheng Xu	Sun Yat-sen University, China
Danping Yang	East China Normal University, China

Invited Speakers

Chuanmiao Chen	Hunan Normal University, China
Jin Cheng	Shanghai University of Finance and Economics, China
Weinan E	Princeton University, USA and Peking University, China
Jialin Hong	Chinese Academy of Sciences, China
Jun Hu	Peking University, China
Jianguo Huang	Shanghai Jiao Tong University, China
Chi-Tien Lin	Providence University, Taiwan
Heping Ma	Shanghai University, China
Jie Shen	Purdue University, USA and Xiamen University, China
Yangfeng Su	Fudan University, China
Jiachang Sun	Institute of Software Chinese Academy of Sciences, China
Tao Tang	Southern University of Science and Technology, China
Li-Lian Wang	Nanyang Technological University, Singapore
Yaushu Wong	University of Alberta, Canada
Roderick S. C. Wong	City University of Hong Kong, Hong Kong
Xuejun Xu	Tongji University, China
Yuesheng Xu	Sun Yat-sen University, China
Danping Yang	East China Normal University, China
Zhimin Zhang	Wayne State University, USA and Beijing Computational Science Research Center, China

Accommodation

All participants will be hosted at the following hotels:

- **Foreign Guest House**（上海师范大学外宾楼，校内）
  Address: No. 100, Guilin Road, Shanghai (徐汇区桂林路 100 号)
  Tel: 021-64322244

- **Yitel Shanghai in Xuhui District**（上海徐汇和颐至尊酒店）
  Address: No. 124 Caobao Road, Shanghai (徐汇区漕宝路 124 号，近桂林路)
  Tel: 021-34019797

Program

Notice:

1. All lectures are held in the Lecture Hall on 2nd Floor of the Yitel Shanghai, No. 124 Caobao Road (上海徐汇和颐至尊酒店，二楼和颐厅，上海徐汇区漕宝路 124 号)
2. Each 45-minute time slot allows 40 minutes for the talk and 5 minutes for questions.

	June 30	July 1	July 2	July 3
Time	Saturday	Sunday	Monday	Tuesday
8:30 ~ 9:15	**Registration on-site 13:30-20:00** (上海徐汇和颐至尊酒店，漕宝路 124 号)	Opening Ceremony Group photo	**Chair: Jie Shen** Chuanmiao Chen Jianguo Huang	**Symposium on Professor Ben-yu Guo's Academic Ideology** (上海师范大学数学系 3 号楼 332 报告厅，桂林路 100 号)
9:15~10:00		**Chair: Zhongci Shi** Heping Ma		
10:00~10:15		**Tea Break**	**Tea Break**	
10:15~11:45		**Chair: Benqi Guo** Roderick S. C. Wong Jiachang Sun	**Chair: Jianguo Huang** Danping Yang Yangfeng Su	
12:00		**Lunch**	**Lunch**	**Lunch**
14:30~16:00		**Chair: Jin Cheng** Jie Shen Jialin Hong	**Chair: Heping Ma** Xuejun Xu Chi-Tien Lin	
16:00~16:15		**Tea Break**	**Tea Break**	
16:15~17:45		**Chair: Jialin Hong** Jin Cheng Jun Hu	**Chair: Xuejun Xu** Li-Lian Wang Yaushu Wong	
18:00		**Dinner**	**Dinner**	

Daily Program

Saturday, June 30, 2018	
13:30-20:00	**Registration on-site:** Yitel Shanghai, No. 124 Caobao Road (上海徐汇和颐至尊酒店，上海徐汇区漕宝路 124 号)

Sunday, July 1, 2018		
8:30 - 9:15	Opening Ceremony, Group photo	
	Session 1, Chair: Zhongci Shi	
9:15 - 10:00	Heping Ma	A Review to the Academic Achievements of Professor Ben-yu Guo
10:00 - 10:15	**Tea Break**	
	Session 2, Chair: Benqi Guo	
10:15 - 11:00	Roderick S. C. Wong	Orthogonal Polynomials and Asymptotic Methods
11:00 - 11:45	Jiachang Sun	On Maltivariable Conforming Elements of n order Laplacian Dirichlet problem in R^d with Simplex Partition
12:00	**Lunch**	
	Session 3, Chair: Jin Cheng	
14:30 - 15:15	Jie Shen	Log orthogonal functions and their applications to fractional PDEs
15:15 - 16:00	Jialin Hong	Numerical Analysis on Ergodic Limit of Approximations for Stochastic NLS Equation
16:00 – 16:15	**Tea Break**	
	Session 4, Chair: Jialin Hong	
16:15 - 17:00	Jin Cheng	The Unique Continuation and its Numerical Treatments
17:00 - 17:45	Jun Hu	TBA
18:00	**Dinner**	

Monday, July 2, 2018		
	Session 1, Chair: Jie Shen	
8:30 - 9:15	Chuanmiao Chen	Riemann 猜想分析，1. —关于抵消原理和高阶公式
9:15 - 10:00	Jianguo Huang	The generalized Arrow-Hurwicz method with applications to fluid computation
10:00 - 10:15	**Tea Break**	
	Session 2, Chair: Jianguo Huang	
10:15 - 11:00	Danping Yang	Wellposedness of variable-coefficient conservative fractional elliptic differential equations and numerical methods
11:00 - 11:45	Yangfeng Su	Theory and Computation of 2D Eigenvalue Problems
12:00	**Lunch**	
	Session 3, Chair: Heping Ma	
14:30 - 15:15	Xuejun Xu	Local Multigrid for the Biharmonic Problem
15:15 - 16:00	Chi-Tien Lin	On pricing Asian options via PDEs with one-state variable
16:00 – 16:15	**Tea Break**	
	Session 4, Chair: Xuejun Xu	
16:15 - 17:00	Li-Lian Wang	Recent Advances in Jacobi Spectral Methods: A Continuation of Guo's Contributions
17:00 - 17:45	Yaushu Wong	Is pollution effect avoidable for numerical solution of Helmholtz equation?
18:00	**Dinner**	

Tuesday, July 3, 2018	
9:00 -12:00	**Symposium on Professor Ben-yu Guo’s Academic Ideology** （上海师范大学数学系 3 号楼 332 报告厅）
12:00	**Lunch**

Abstracts of Talks

Riemann 猜想分析，1.
—关于抵消原理和高阶公式

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1859 年 Riemann 取复数 $s=it+\sigma$，绕过极点 $s=1$ 解析延拓 $\zeta(s)$ 到全平面，得到

$$\begin{cases}\xi(s)=\int_1^{\infty}(x^{s/2-1}+x^{-s/2-1/2})(2x^2\psi''(x)-3x\psi'(x))dx\,,\\ \psi(x)=\sum_{n=1}^{\infty}e^{-a_n x}\ ,\ a_n=n^2\pi\,,\end{cases} \tag{1}$$

它在临界线 $\sigma=1/2$ 上是实函数. 他提出了以下命题，

Riemann 假设（RH）. 在临界区域 $\Omega=\{s=\sigma+it:0\le\sigma\le 1,0<t<\infty\}$ 中，$\xi(s)$ 函数的零点都在临界线 $\sigma=1/2$ 上（称为平凡零点）.

这是一个极其困难的问题，也可认为是一个反问题. 至今在理论分析和大规模计算两个方面已经作了大量工作，都认为 RH 成立，但未能解决.

问题出在哪里？我们引用 3 个学者的话：Conrey 说，“In my belief, RH is a genuinely arithmetic question that likely will not succumb to methods of analysis”. Edwards 指出“.…about the function ξ which are not available to us today”. Selberg 指出“There have probably been very few attempts at proving the Riemann conjecture, because, simply, no one had ever had any really good idea for how to go about it”. 简言之，许多估计是利用 $\zeta(s)$ 得到的，但 ζ 是发散级数.至今没有用到 $\xi(s)$. 证明 RH 还不知道如何下手.

按我的理解，第一个困难，Jacobi 函数 $\psi(x)$ 是一个无穷级数，至今没有简单的解析表示. 指数衰减长 $|\xi(s)|\le Ct^3e^{-t\pi/4}$ 是利用 ζ 函数估计的.从 $\xi(s)$ 本身看不出为什么会是指数衰减?

我们按刘微方法论“计算可探测未知性质和方法”，作了如下尝试.

1). 用 Matlab 计算(0,1)，对 $t\le 64$，发现对每个 n，每项是 $O(t^{-1})$，4 项和，实部 $O(t^{-2})$，虚部 $O(t^{-3}\beta)$. 这里 $\beta=\sigma-1/2>0$. 但对 n 求和后变为指数衰减

$O(e^{-t\pi/4})$ 和高频振荡. 因此其中一定“隐藏”着一种“抵消”机制.

2). 沿着 Riemann 思路，发现导数值 $D^j\psi(1)$ 的线性组合满足（称抵消原理）

$$L_m(\psi)=\sum_{j=1}^{m}c_{mj}D^j\psi(1)=0, m=2,3,4,... \tag{2}$$

Rielmann 推导公式(0.1)使用了 $m=1, 1/2+\psi(1)+4\psi'(1)=0$.

3). 利用此抵消原理，推导了高阶公式，对任何偶数 $m\ge 2$，

$$\begin{cases}\xi(s) \quad =\tau^{2-m}W_m(s)\,,\ \tau=(it+\beta)/2\,,\\ W_m(s)=\int_0^\infty(x^{s/2-1}+x^{-s/2-1/2})F_m(x)dx\,,\\ F_m(x)=\sum_{j=1}^{m}C_{mj}x^jD^j\psi(1)\,,\end{cases} \tag{3}$$

$m=2$ 就是公式(0.1)，而系数 C_{mj} 由递推公式得到，它们随 m 增大而快速增长.由于 $\left|\tau^{2-m}\right|>0$，由此得到一个“等价性”：由 $\left|W_m(s)\right|>0$ 可推出长 $\xi(s)>0$. 这是一个变形公式，因子 $\tau^{2\text{-}m}$ 快速减小，其研究仍极其困难.

记 $\tau=s/2$，$\tau'=(1-s)/2$，$\tau+\tau'=1/2$. 我们转向等价于 $\xi(s)$ 的函数

$$\eta(s)=\frac{2}{s(1-s)}\xi(s)=\frac{1}{s(1-s)}-\int_1^\infty(x^{\tau-1}+x^{\tau'-1})\psi(x)dx=W(s)-G(s) \tag{4}$$

并考虑它的有限和 $\eta^N=W^N-G^N$，

$$\begin{cases}G^N(s)=\sum_{n=1}^{N}\int_0^\infty(x^{\tau-1}+x^{\tau'-1})e^{-a_nx}dx\,,\\ \qquad\quad =\sum_{n=1}^{N}(a_n{}^{-\tau}\Gamma(\tau)+a_n{}^{-\tau'}\Gamma(\tau'))\,,a_n=n^2\pi\,,\\ W^N(s)=\dfrac{1/4}{\tau\tau'}+\sum_{n=1}^{N}\int_0^1(x^{\tau-1}+x^{\tau'-1})e^{-a_nx}dx\,,\end{cases} \tag{5}$$

这里 W^N 可以展开为 τ,τ' 的分式函数收敛级数（抵消原理有效）.

对 $t\le 64$ 及 $\beta\le 0.5$ 的计算表明，$\left|W^N\right|$，$\left|G^N\right|$ 都有标准指数衰减,且为正. 由于 W^N，G^N 是同阶量，也有 $\left|W^N-G^N\right|>0$，但不可能由 W^N，G^N 的正性证明. 此结构很糟！

上述试验有两点教训：1). $G^N(s)$ 用 ζ 函数表示，背离了 Rtemam 解析延拓思想，我

们应将$\psi(x)$看作是一个解析函数处理；2). 将希望寄托在高阶公式，是否改变上述结构，这是以后的工作.

The Unique Continuation and its Numerical Treatments

Jin Cheng
Shanghai University of Finance and Economics

The unique continuation property means that the solution of the partial differential equations on the small domain can determine the solution on the large connected domain. This is useful for study of inverse problems for partial differential equations. In this talk, we discuss the problems of determining the solutions of the elliptic equation from the partial information of the solution on the small domain. The stable numerical algorithms are proposed based on the conditional stability for the unique continuation problems.

Numerical Analysis on Ergodic Limit of Approximations for Stochastic NLS Equation

Jialin Hong
LSEC, Institute of Computational Mathematics and Scientific/Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences
Email: hjl@lsec.cc.ac.cn

In this talk we present some results on numerical analysis of ergodic limit of approximations for stochastic nonlinear Schroedinger (NLS) equation. The equation considered is charge conservative and has the multi-symplectic conservation law. By constructing control system and invariant control set, it is proved that a finite dimensional approximation, based on the finite central difference scheme, possesses a unique invariant measure on the unit sphere. Furthermore, the midpoint scheme is applied further to get a full discretization, which possesses the discrete charge conservation law and the discrete multi-symplectic conservation law. Utilizing the Poisson equation corresponding to the finite dimensional approximation, the convergence error between the temporal average of the full discretization and the ergodic limit of the finite dimensional approximation is derived (In collaboration with Dr. Xu Wang and Dr. Liying Zhang).

The generalized Arrow-Hurwicz method with applications to fluid computation

Jianguo Huang
School of Mathematical Sciences, Shanghai Jiao Tong University,
Shanghai 200240, China

In this talk, we will first discuss the existence and uniqueness of a class of nonlinear saddle-point problems, which are frequently encountered in industrial and engineering applications. Then, a generalized Arrow-Hurwicz method is introduced to solve such problems. For the method, the convergence rate analysis is established under some reasonable conditions. It is also applied to solve three typical discrete methods in fluid computation, with the computational efficiency demonstrated by a series of numerical experiments. This is a joint work with Binbin Du from Shanghai Jiao Tong University.

On pricing Asian options via PDEs with one-state variable

Chi-Tien Lin
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Email: ctlin@pu.edu.tw

Asian options, also called average options, have a terminal payoff depending on some form of averaging of the underlying asset. The averaging feature can lessen incentives for market manipulation and as the volatility of an average is lower than the volatility of the underlying asset, they are useful in risk management. They therefore tend to be popular with corporate treasurers.

In this talk, we consider fixed and floating strike European style Asian call and put options. For such options, there is no convenient closed-form formula for the prices. Previously, Rogers and Shi, Vecer, and Dubois and Lelièvre have derived partial differential equations with one state variable, with the stock price as numeraire, for the option prices. In this talk, we derive a whole family of partial differential equations, each with one state variable with the stock price as numeraire, from which Asian options can be priced. Any one of these partial differential equations can be transformed into any other. This family includes four partial differential equations which have a particularly simple form including the three found by Rogers and Shi, Vecer, and Dubois and Lelièvre. Recently, Vecer derive a new PDE using the average asset as numeraire. We perform numerical comparisons of the five partial differential equations by Crank-Nicolson method and conclude, as expected, that Vecer's equations and that of Dubois and Lelièvre do better when the volatility is low but that with higher volatilities the performance of all five equations is similar. Vecer's equations are unique in possessing a certain martingale property and they perform numerically well or better than the others.

Joint work with C. BROWN (1), J. C. HANDLEY (2) and K. J. PALMER (3)

(1) Monash University, Caulfield, Victoria, Australia,

(2) University of Melbourne, Parkville, Victoria, Australia,

(3) National Taiwan University, Taiwan, retired.

郭本瑜教授学术成就回顾

马和平

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今年 5 月 5 日是著名数学家郭本瑜教授逝世二周年，我们深切缅怀郭本瑜教授一生的事业，回顾郭本瑜教授的学术成就。郭本瑜教授对科学计算的方法、理论及其应用作出了多方面的杰出贡献，在国际学术界享有盛誉。他在 20 世纪 60 年代就率先提出从保持原问题整体性质出发设计计算流体力学问题的差分格式，具有前瞻

性；他是谱方法研究的开拓者，是国际上最早系统研究谱方法的专家之一，对谱方法及其应用作出了系统和独特的贡献；他独立设计了数值天气预报的平方守恒算法和谱模式，比著名气象学家 A. Arakawa 的工作更完美，是当时重要的前沿性成果；他基于能量守恒等物理原理构造了卓有成效的非线性波计算方法，并揭示了若干边界脉冲诱发孤波现象；他提出非线性问题逼近的广义稳定性及其分析方法，推广了著名数学家 P. D. Lax 和 Л. В. Канторοъич 的有关理论，为逼近非线性问题的稳定性研究发展了一套强有力的框架和一系列工具；他还在非可积系统 Painlevé 分析方法和生物数学等领域得到十分重要的结果。

Log orthogonal functions and their applications to fractional PDEs

Jie Shen
Department of Mathematics, Purdue University,
School of Mathematical Sciences, Xiamen University
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It is well known that solutions of time-fractional ODEs/PDEs exhibit weak singularities at $t=0$ so that usual approximation techniques do not yield accurate results.

We construct two new classes of orthogonal functions, the log orthogonal functions (LOFs) and the generalized log orthogonal functions (GLOFs) by applying a log mapping to the Laguerre functions. We develop basic approximation theory for these new orthogonal functions, and show that they are particularly suitable for functions which have weak singularities at one endpoint. In particular, for functions involving one or multiple terms of t^r ($r \ge 0$ but not integers), which are present in typical solutions of time-fractional differential equations, their approximations by GLOFs will converge exponentially, as opposed to a low algebraic rate if usual orthogonal polynomials are used.

We then use the GLOFs in time and any suitable approximation in space to construct space-time Petrov-Galerkin method for time-fractional PDEs.

However, due to the Petrov-Galerkin approach in time, the linear system cannot be accurately solved by using the usual matrix-diagonalization method; we shall propose a novel approach to overcome this difficulty, so that our space-time method for time-fractional PDEs is accurate as well as efficient. Ample numerical results will be presented to demonstrate the effectiveness of our methods.

Theory and Computation of 2D Eigenvalue Problems

Yangfeng Su
Fudan University

The 2D eigenvalue problem (2dEVP) is a class of the double eigenvalue problems first studied by Blum and Chang in 1970s. The 2dEVP seeks scalars $\lambda, \mu,$ and a corresponding vector x satisfying the following equations

$$Ax = \lambda x + \mu Cx \ ,$$
$$x^H Cx = 0 \ ,$$
$$x^H x = 1,$$

where A and C are Hermitian and C is indefinite. We show the connections between 2dEVP with well-known numerical linear algebra and optimization problems such as quadratic programming, the distance to instability and H_∞-norm We will discuss (1) fundamental properties of 2dEVP including well-posedness, types and regularity, (2) numerical algorithms with backward error analysis.

On Maltivariable Conforming Elements of n order Laplacian Dirichlet problem in R^d with Simplex Partition

Jiachang Sun
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In this talk we investigate B-spline function space as s kind of conforming finite element for general n order Laplacian Dirichlet problem in R^d as

$$\begin{cases} -\Delta^n u = f, \\ D^\nu u\big|_{\partial\Omega} = 0, \nu = 0,1,...n-1. \end{cases} \qquad (1)$$

where $\Omega \in R^d$ can be decomposed of a $d+1$-direction simplex partition Ω_Δ, and B-spline are the conforming space with minimum support among the d+1-direction simplex partition.

Let $\prod_k (R^d)$ be the space of all polynomials over Ω with total degree $\leq k$ in R^d. In general, $S^{2m}_{(d+1)m}(\Omega)$ and $S^{2m}_{(d+1)m+1}(\Omega)$ are the conforming B-spline space with minimal degree for high order Laplace equation (1) with $n = 2m-1$ and $n = 2m$, respectively. There are two main arguments in the talk:

Theorem 1

$$\dim(S^{2m-1,2m-1}_{(d+1)m}(\Omega)) := T_{\Omega_{-m}} \qquad (2)$$

Equals to the number of internal rector in the domain Ω_{-m}. *Moreover, there are* $d(d+1)!\sum_{j=1}^{m} j^{d-1} + 1$ *unit simplex in the support of B-spline in* $S^{2m-1,2m-1}_{(d+1)m}(\Omega)$ *,where* Ω_{-m} *is the m-order contractive domain of* Ω.

Theorem 2

$$\dim(S^{2m,2m}_{(d+1)m+1}(\Omega)) := V_{\Omega_{-m}} \qquad (3)$$

Equals to the number of internal vertex in the domain Ω_{-m}. *Moreover, there are* $(d+1)m(m-1)^{d-1} + 1$ *vertex in the support of B-spline in* $S^{2m,2m}_{(d+1)m+1}(\Omega)$.

Recent Advances in Jacobi Spectral Methods: A Continuation of Guo's Contributions

Li-Lian Wang
Division of Mathematical Sciences,
School of Physical and Mathematical Sciences,
Nanyang Technological University

Among late Professor Ben-yu Guo's many contributions to spectral methods, the development of Jacobi spectral methods is deemed to be the most significant one. His first paper along this line was published in 1998. I was very fortunate to work with him on this topic since my PhD study. Given this event, I feel compelled to memorize his contributions, and more importantly, continue his seminal works.

In this talk, we briefly review Jacobi approximations and Jacobi spectral methods with emphasis on Prof. Guo's works. We then introduce new results under the framework of fractional calculus (FC). We show that how FC can radically impact spectral approximation theory and singularity analysis. Moreover, we discuss Jacobi spectral methods using non-standard basis functions for PDEs with nonlocal operators.

Is pollution effect avoidable for numerical solution of Helmholtz equation?

Yau Shu Wong
Department of Mathematical and Statistical Sciences
University of Alberta
Edmonton, Alberta, Canada

Helmholtz equation arises in problems related to wave propagations, such as acoustic, electromagnetic wave scattering and in many geophysical applications. Developing efficient and highly accurate numerical approximation for solving the Helmholtz equation at large wave numbers is a very challenging computational task and it has attracted a great deal of attention for a long time. The foremost difficulty in solving Helmholtz equation numerically is to eliminate or minimize the pollution effect which could lead to a serious problem as the wave number increases. Let k, h and n denote the wave number, the grid size and the order of a numerical scheme, it can be showed that due to the pollution effect, the computational error for many existing numerical solutions is proportional to $k\,(kh)^n$. It has been reported that it is impossible to eliminate the pollution effect for problems in two and three dimensions. Another difficulty is that the resulting linear system is indefinite and many efficient iteration schemes can not be applied. In this talk, we present numerical algorithms based on global difference approximations. The most attractive feature of the developed numerical schemes is that they are pollution free and the error is bounded by Ch^n, where C is a positive constant. Numerical simulations for multi-dimensional Helmholtz equation at high wave numbers will be reported.

Orthogonal Polynomials and Asymptotic Methods

Roderick S. C. Wong

City University of Hong Kong, Hong Kong

Orthogonal Polynomials is an important classical subject in mathematics, physics and engineering. One of the research topics in this area is to investigate the behaviours of these polynomials as their degree tends to infinity. In this talk, I will first recall some discrete and neo-classical orthogonal polynomials. Then I will briefly describe various asymptotic methods in differential equations that are available for applications. Finally, I will present a recent development in the asymptotic theory for second-order difference equations. The importance of this development is that orthogonal polynomials may not satisfy any differential equations, but they all satisfy a three-term recurrence relation, which is a second-order difference equation.

Local Multigrid for the Biharmonic Problem

Xuejun Xu
School of Mathematical Sciences, Tongji University
and
Institute of Computational Mathematics, AMSS,CAS, Beijing

In this talk, we shall present some local multigrid methods (LMG) to solve the linear algebraic systems resulting from the application of adaptive conforming plate element and nonconforming plate element approximations to the biharmonic problem. The abstract Schwarz framework is applied to verify the uniform convergence of the local multilevel methods featuring Jacobi and Gauss-Seidel smoothing only on local nodes associated with the local refinements. By the abstract framework, convergence estimate may also be derived from the stability of the space splitting and its strengthen Cauchy-Schwarz inequality.

Wellposedness of variable-coefficient conservative fractional elliptic differential equations and numerical methods

Danping Yang
Department of Mathematics
East China Normal University
Shanghai, 200241, China

Fractional diffusion equations describe various phenomena exhibiting anomalous diffusion that cannot be modeled accurately by second-order diffusion equations. Fractional differential equations raise many mathematical difficulties that have not been encountered in the analysis of second-order differential equations.

In this talk, we discuss the existence and uniqueness of the solution of fractional differential equations of Dirichlet and fractional Newman boundary conditions. At the first part, we present a counterexample which shows that the Galerkin weak formulation loses coercivity in the context of variable-coefficient conservative fractional elliptic differential equations. Hence, the previous results cannot be extended to variable-coefficient conservative fractional elliptic differential equations. We adopt an alternative approach to prove the existence and uniqueness of the classical solution to the variable-coefficient conservative fractional elliptic differential equation and characterize the solution in terms of the classical solutions to second-order elliptic differential equations. Furthermore, we derive a Petrov-Galerkin weak formulation to the fractional elliptic differential equation. We prove that the bilinear form of the Petrov-Galerkin weak formulation is weakly coercive and so the weak formulation has a unique weak solution and is well posed. Finally, we outline potential application of these results in the development of numerical methods for variable-coefficient conservative fractional differential equations. At the second part, we consider the fractional Newman boundary value problems. We shall see there exist many kind of fractional Newman boundary value conditions, for different Newman boundary value conditions, in some cases there exists the solutions but in others there are not solution.

Map

A: Yitel Shanghai, No. 124 Caobao Road (上海徐汇和颐至尊酒店，上海徐汇区漕宝路 124 号)

B: Foreign Guest House of Shanghai Normal University （上海师范大学外宾楼）

C. Department of Mathematics, Shanghai Normal University（上海师范大学数学系）

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